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The Effect of the Geographical Distribution of Observations on Orbital Accuracy

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The effect of the geographical distribution of observations on orbital accuracy

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An Earth satellite can usually be observed by optical methods only on two fairly small arcs of its orbit. At present, most observations are made on only one of these arcs.

In order to investigate the improvement in orbital accuracy which would be obtained if both arcs were fully observed, the orbit of Echo 1 rocket (1960ι2) has been determined from field-reduced Baker-Nunn observations, using (i) only Northern Hemisphere observations, (ii) only Southern Hemisphere observations, and (iii) both sets together. The accuracy of e, i, Ω and ω may be improved by a factor of about 4 by using observations from both Hemispheres.

1. Introduction

Some of the satellite orbits so far determined at the Royal Aircraft Establishment have been limited in accuracy because the visual observations available have been confined to a small arc of the orbit.

The observational requirements for maximum accuracy in an orbit determination are different for the six orbital elements: the accuracy of the semi-major axis, a, is governed by the spread of the observations in time, since a is derived from the orbital period. Accurate values of the inclination i are obtained from observations at apex; similarly, Ω, the right ascension of the node, and t_0, the time of the nodal crossing, are best determined if observations are available near the equator. To obtain good values of the eccentricity e and the argument of perigee ω, we would require groups of observations at intervals of about 90° round the orbit. An ideal situation therefore would be to have groups of observations at northern and southern apex, and at both equatorial crossings, although three of these groups, omitting for instance southern apex, would probably still be adequate for most orbits.

Unfortunately this situation is not achievable when only optical observations are available, since there are physical limitations on the orbital coverage: first, about half the orbit will lie over the sunlit side of the Earth, and secondly, the satellite will usually pass through the Earth's shadow; in both these conditions it is impossible to observe the satellite by optical methods. If we require complete coverage of the orbit we must resort to radio and radar observations.

The purpose of this paper is to investigate the improvement in accuracy which could be obtained if full advantage were taken of the parts of the orbit where a satellite is visible.

The Baker-Nunn camera network, which consists of twelve cameras, eight in the Northern Hemisphere and four in the Southern, provides good coverage between latitudes 40° N and 40° S. Since Baker-Nunn field-reduced observations are of a similar directional accuracy to those made by amateur visual observers, we decided to use some of these to determine an orbit, and compare the accuracies obtained using observations from just one Hemisphere (an approximation to the usual situation) and from both Hemispheres.

In order to avoid some of the incidental problems of orbital determination, we decided to choose a drag-free orbit with a small (but not too small) eccentricity. Also, since the Baker-Nunn cameras are all situated between latitudes 40° N and 40° S, a satellite with an inclination much greater than 40° would be beyond the range of the cameras at apex. We therefore chose Echo 1 rocket (1960 ι2), which has an inclination of 47°, an eccentricity of 0·01, and is in orbit about 1600 km above the Earth's surface.

Although the timing accuracy of the Baker-Nunn observations is about 50 times better than that of stopwatch-timed observations, timing errors are rendered unimportant by the choice of a satellite in a high circular orbit.

A suitable period of observations (S.A.O. 1963) from both Hemispheres occurred in July 1962, and the Smithsonian Astrophysical Observatory have also determined the orbit for the same period from precisely reduced observations (S.A.O. 1964).

2. Results

The orbital elements were calculated at five ascending nodes, at intervals of 25 revolutions. For convenience the node at 21 h 06 m on 9 July 1962 was designated Node 0. The orbits were determined three times, first using observations from the Northern Hemisphere only, second using Southern Hemisphere observations only, and finally using all the observations. The six parameters fitted by the least-squares process were t_0, a, e, i, Ω and ω. The orbital parameters are defined in the Appendix, and the values obtained are given in table 1.

An interesting point to note is the value of ϵ, which represents an *a posteriori* estimate of the standard deviation of the observations: ϵ has an average value of 0·4, which implies that the observations have an accuracy of about 1·2′, instead of the value of 3′ recommended by the S.A.O.

The orbital coverage is defined here by the quantity C, which shows the percentage of the orbit covered by observations; the symbol (2) after the value of C indicates that the observations lie in two distinct arcs with a gap of 10 % or more between. The coverage is illustrated in figure 1, which shows the positions round the orbit of the observations used in each determination. The positions are shown as fractions of a revolution, with zero corresponding to the equatorial crossing. It can be seen that for nodes 0 to 50 the observations are divided into two distinct bands north and south of the equator, whereas they form one continuous band for the last two nodes; this is because the satellite entered an all-in-sunlight period towards the end of the time we are considering.

Figure 1 shows the observations used in the final determination, using all observations within about $1\frac{1}{2}$ days on either side of the working node. The observations used in the 'Northern Hemisphere only' determination are those above the axis in figure 1, with one exception: this was an observation made from Curaçao (which is just north of the equator) when the satellite was south of the equator. Some of the 'Southern Hemisphere' determinations include more observations than those shown below the axis in figure 1 under a particular node, since these were insufficient for an orbit determination. The dashed curves in figure 1 show approximate limits of visibility of the satellite as deduced from the extent of the observations.

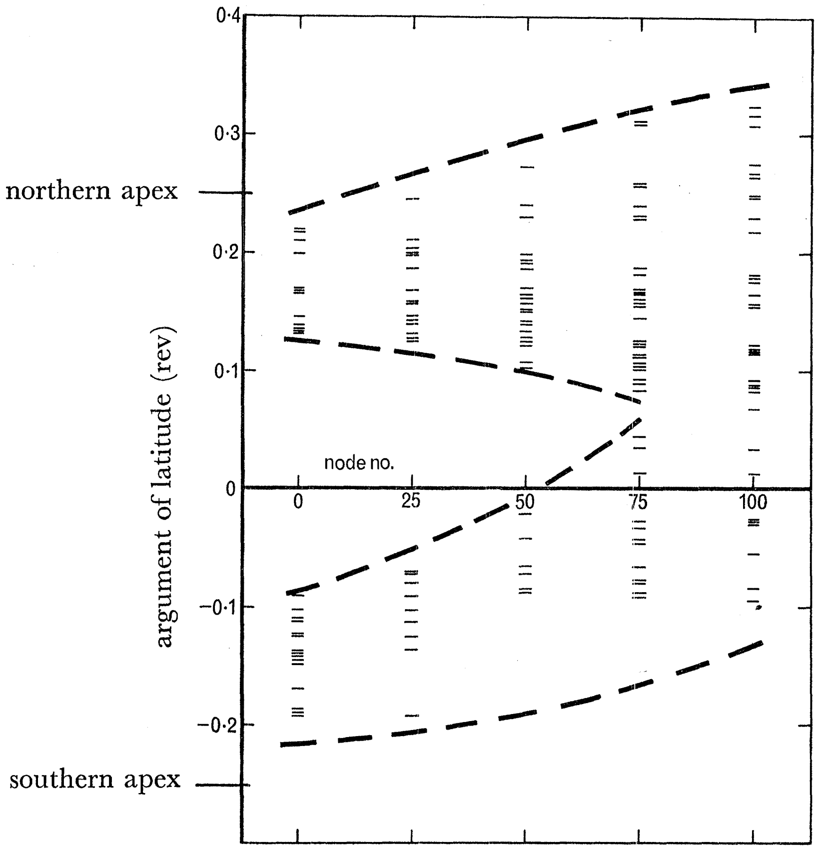

FIGURE 1. Distribution of observations around orbit.

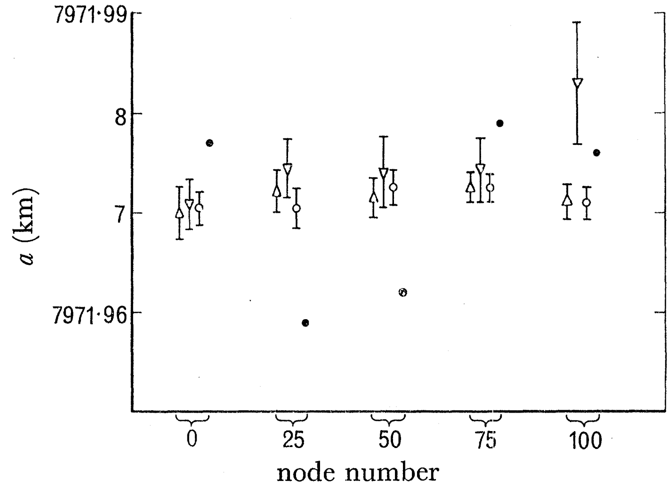

FIGURE 2. Values of semi-major axis for Echo 1 rocket. △, N. Hemisphere; ▽, S. Hemisphere; ○, all observations; ●, S.A.O. orbit.

Figures 2 to 6 show a comparison of the different sets of values of the orbital elements. The vertical bars represent the standard deviations of the R.A.E. values; the four values for each node have been separated horizontally for clarity, although they do in fact refer to the same time. The S.A.O. orbital elements (S.A.O. 1964), have been converted to the R.A.E. definitions using the equations given by Merson (1963 *a*). The standard deviations

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of the S.A.O. elements are not shown, since the values were obtained by interpolation; the quoted s.d.'s are in general about one-tenth of ours. The S.A.O. orbit was obtained from precisely reduced observations, which are about 20 times more accurate than the field-reduced.

Figure 2 shows the values of semi-major axis a; all the R.A.E. values are in close agreement, and this parameter is the least affected by the geographical distribution of the

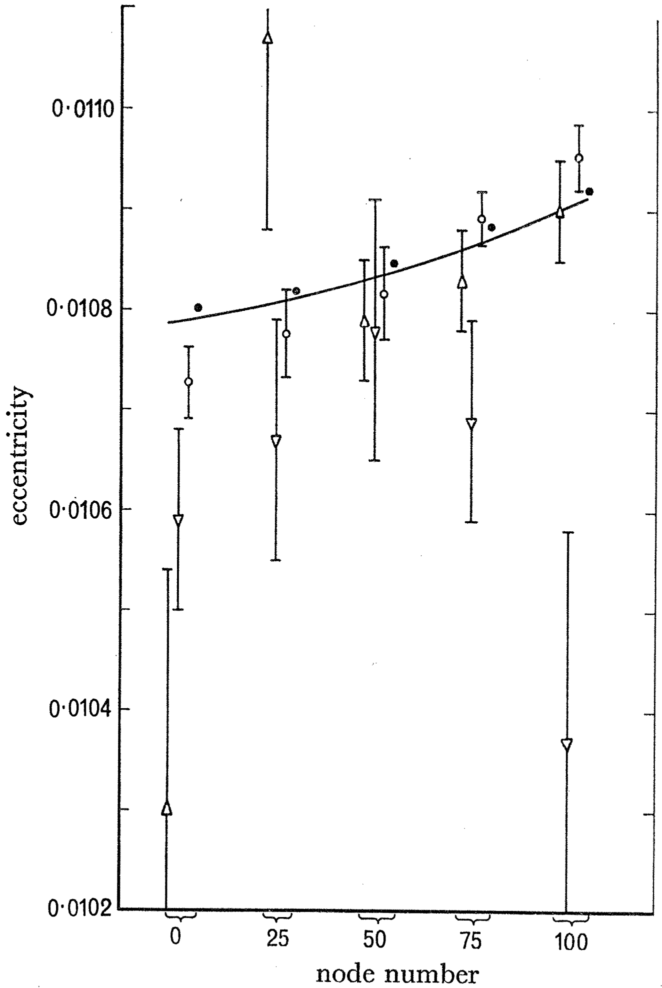

FIGURE 3. Values of eccentricity for Echo 1 rocket. △, N. Hemisphere; ▽, S. Hemisphere; ○, all observations; ●, S.A.O. orbit; ——, theoretical variation.

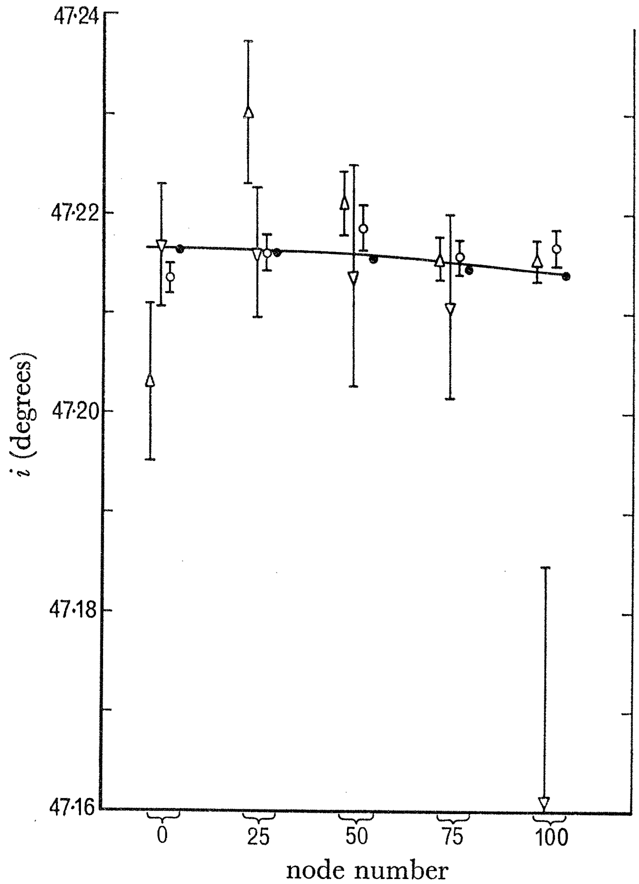

FIGURE 4. Values of inclination for Echo 1 rocket. △, N. Hemisphere; ▽, S. Hemisphere; ○, all observations; ●, S.A.O. orbit; ——, theoretical variation.

observations, since it is calculated from the satellite's period and depends more on the distribution of the observations in time. The reason for the difference between the R.A.E. and S.A.O. values is unknown: a similar variation occurs in the S.A.O. values of n, so the discrepancy cannot be entirely due to the rounding errors which will inevitably occur in converting a to the R.A.E. definition.

The values of eccentricity e are shown in figure 3. The standard deviations of the values derived from all observations are a quarter to a third of those for the other sets of values;

for the Northern Hemisphere set, the standard deviation decreases by a factor of 6 as the coverage increases from 10 to 30%.

The continuous curve in figure 3 shows $\bar{e}$, the theoretical variation of e. The shape of this curve is determined by integrating de/dt; the constant $\bar{e}_0$ which fixes the position of the curve was found by a least-squares fit to the values of e obtained from all observations. Most of the values of e lie within two standard deviations of the curve.

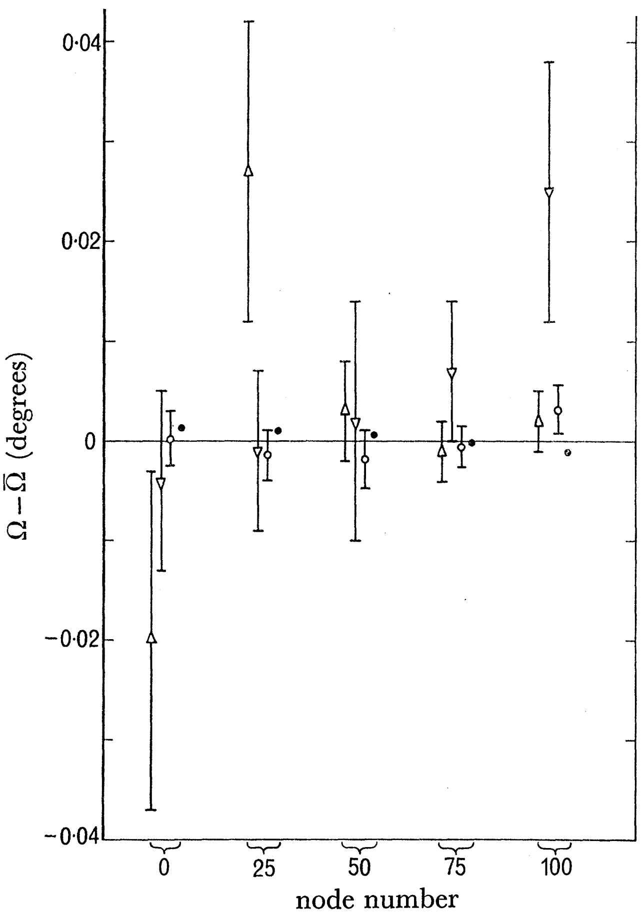

FIGURE 5. Residuals in Ω for Echo 1 rocket. △, N. Hemisphere; ▽, S. Hemisphere; ○, all observations; ●, S.A.O. orbit.

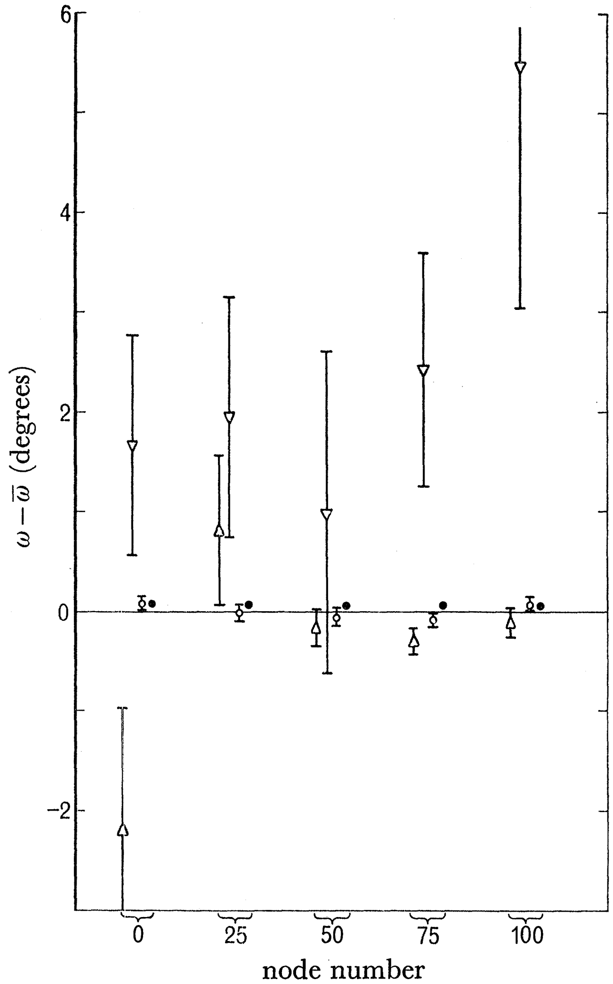

FIGURE 6. Residuals in ω for Echo 1 rocket. △, N. Hemisphere; ▽, S. Hemisphere; ○, all observations; ●, S.A.O. orbit.

The S.A.O. values follow the shape of the smoothed curve but are displaced from it, suggesting that our value of $\bar{e}_0$ is about 0·00001 too low; since $\sigma(\bar{e}_0) = 0{\cdot}00002$, this is excellent agreement.

Figure 4 shows the values of the inclination i, together with its theoretical variation calculated in the same way as for e. Once again the smoothed curve and the S.A.O. values agree very well, the difference being about 0·0004°, while $\sigma(\bar{i}_0)$ is about 0·001°.

All the R.A.E. values, with one exception, lie within two standard deviations of the theoretical curve. The standard deviations of the Northern Hemisphere values are generally better than those for the Southern Hemisphere: this is due not only to the better coverage, but also to the fact that the Northern Hemisphere observations include some near northern apex (see figure 1). If we ignore the one exceptional value, the average s.d. for a coverage of around 10% is 0·008°, which decreases to about 0·002° for a coverage of 20%. For still greater coverage, the standard deviation remains near this value, which suggests that the accuracy has reached a limit which can probably only be exceeded by having more accurate observations.

The values $\overline{\Omega}$ obtained from the theoretical variation (−3·1°/day) have been subtracted from the observed values of Ω and the residuals are shown in figure 5. The consistency of all the sets of values is very good. The standard deviations of the values obtained from all observations are a factor of 4 lower than those for the two hemispheres taken separately. As for i, 0·002° seems to be a lower limit to the standard deviation.

The values of ω, the argument of perigee, with the theoretical variation (3·1°/day) subtracted out, are shown in figure 6. ω is the least accurate of the orbital elements, because of the small eccentricity. However, there is a dramatic improvement in the accuracy when observations from both Hemispheres are used. The average value of $e\sigma(\omega)$ for single-Hemisphere coverage is 0·01°; this value is reduced to less than 0·001° in the final orbit.

3. Conclusions

By analysing the orbit of Echo 1 rocket in three different ways it has been shown that the use of observations from both hemispheres instead of just one considerably improves the accuracy of the eccentricity and the three angular parameters. Thus, if we could supplement the usual observational coverage of 10% of the orbit in the Northern Hemisphere by a similar band in the Southern Hemisphere, we might expect reductions in the standard deviations of e by a factor of about 3, of i and Ω by a factor of 4, and of ω by a factor of 10, although this last may be rather an extreme value, resulting from the low eccentricity. For more eccentric orbits, it might be reasonable to expect improvement by a factor of 4, as for the other elements.

The semi-major axis and the mean motion, from which it is derived, are uninfluenced by the geographical distribution of the observations. The accuracy of the other elements is strongly dependent on the orbital coverage, as well as being subject to the special requirements listed in the introduction.

The limited orbital coverage obtained in the past has been a result of the concentration of observers in northwest Europe and North America, and the fact that, during any one period of observations, a satellite is usually observed on either north-bound or south-bound passes alone. The accuracy of future orbit determinations from visual observations could be much improved if the orbital coverage were extended by (i) increasing the number of observers in the Southern Hemisphere; (ii) obtaining observations at apex whenever a satellite's inclination is suitable; and (iii) observing both north- and south-bound transits whenever both are visible. One should be cautious about drawing too firm conclusions from a single orbit, but the results derived here have already been confirmed by a recent determination (Scott 1967) of the orbit of Injun 3 rocket. The inclusion of just a few observations from one station in Australia resulted in the reduction of the standard deviations of e, i, Ω and ω by a factor of between 2 and 3.

The results obtained here give increased weight to the Cospar (1966) recommendation 'that every effort should be made to increase the numbers and improve the geographical distribution of visual observations of low-perigee satellites and especially to secure more observations from the southern hemisphere'.

Appendix. Notes on the orbital parameters

In table 1, the first column gives the node number relative to the ascending node on 9 July 1962 at 21 h 06 m, which has been taken as zero for convenience. The remaining columns are as follows:

date
time, t_0 (hours, minutes and seconds)
semi-major axis a (km)
eccentricity, e
e_1 (100 days)$^{-1}$
inclination, i (degrees)
i_1 (deg/100 days)
R.A. of the node, Ω (degrees)
Ω_1 (deg/100 days)
argument of perigee, ω (degrees)
ω_1 (deg/100 days)
mean motion, n (deg/100 days)
number of observations used, N
extent of observations, D (days)
ϵ, the standard deviation of an observation of unit weight
C, the percentage of the orbit covered by observations, with the number of arcs into which coverage is divided, if more than one
date and time in Modified Julian Days

The six parameters a, e, i, Ω, ω and t_0 were determined by a least-squares fit to the observations. The figures after the values of these parameters denote the s.d. of the parameter, in units of the final figure quoted. The linear coefficients e_1, i_1, Ω_1 and ω_1 were calculated from theory; although e_1 and i_1 represent the rates of change of e and i (since the orbit is virtually drag free), Ω_1 and ω_1 do not include the secular changes in Ω and ω due to J_2. For a detailed description of the dynamical model and the orbit improvement programme see Merson (1963*b*) and Gooding (1964).

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S.A.O. 1963 *Smithson. Astrophys. Obs. Spec. Rep.* no. 131.
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TABLE 1. ORBITAL PARAMETERS OF ECHO 1 ROCKET

node	1962 July	time h m s	a	e	$10^5 e_1$	i	$10^3 i_1$	Ω	$10^3 \Omega_1$	ω	ω_1	n	N	D	ϵ	C	M.J.D.
								(i) *Northern Hemisphere observations*									
0	9	21 06 04·41 45	7971·970 3	0·01030 24	77	47·2031 79	−20	−103·313 17	165	280·62 122	17·18	439094·1	12	2·46	0·38	8·9	37854·8792177
25	11	22 15 38·16 43	7971·972 2	0·01107 19	111	47·2302 71	− 4	−109·618 15	194	290·08 75	16·53	439094·0	13	2·54	0·27	12·0	37856·9275250
50	13	23 25 10·20 10	7971·972 2	0·01079 6	145	47·2211 32	−31	−115·994 5	206	295·55 18	15·36	439094·0	23	2·62	0·34	17·1 (2)	37858·9758125
75	16	0 34 43·09 6	7971·973 2	0·01083 5	180	47·2155 21	−51	−122·349 3	192	301·84 13	14·59	439093·9	29	2·64	0·35	28·3	37861·0241098
100	18	1 44 16·34 8	7971·971 2	0·01090 5	214	47·2153 20	−33	−128·699 3	163	308·43 14	13·34	439094·0	26	2·38	0·40	31·2	37863·0724113
								(ii) *Southern Hemisphere observations*									
0	9	21 06 04·97 23	7971·971 2	0·01059 9	77	47·2168 61	−20	−103·297 9	165	284·48 110	17·18	439094·0	15	2·71	0·39	10·2	37854·8792242
25	11	22 15 37·33 17	7971·964 3	0·01057 12	111	47·2161 65	− 4	−109·646 8	194	291·22 119	16·53	439094·6	12	3·12	0·41	12·2	37856·9275154
50	13	23 25 10·14 16	7971·974 4	0·01078 13	145	47·2138 111	−31	−115·995 12	206	296·71 160	15·36	439093·8	10	3·78	0·48	12·0	37858·9758118
75	16	0 34 43·18 9	7971·974 3	0·01069 10	180	47·2107 92	−51	−122·341 7	192	304·56 116	14·59	439093·8	11	2·95	0·40	12·6	37861·0241109
100	18	1 44 16·70 18	7971·983 6	0·01037 21	214	47·1611 236	−33	−128·676 13	163	314·01 244	13·34	439093·0	10	3·93	0·57	8·1	37863·0724155
								(iii) *All observations*									
0	9	21 06 05·273 64	7971·970 2	0·010727 36	77	47·2135 15	−20	−103·2925 27	165	282·890 68	17·18	439094·1	27	2·73	0·39	19·1 (2)	37854·8792277
25	11	22 15 37·560 62	7971·970 2	0·010776 43	111	47·2161 18	− 4	−109·6463 25	194	289·260 78	16·53	439094·1	22	2·82	0·37	24·2 (2)	37856·9275181
50	13	23 25 10·171 53	7971·973 2	0·010817 46	145	47·2186 23	−31	−115·9985 29	206	295·660 86	15·36	439093·9	28	2·62	0·35	23·8 (2)	37858·9758122
75	16	0 34 43·173 35	7971·972 1	0·010892 26	180	47·2157 17	−51	−122·3491 20	192	302·061 67	14·59	439093·9	39	2·65	0·37	46·5	37861·0241108
100	18	1 44 16·435 45	7971·971 2	0·010953 32	214	47·2166 18	−33	−128·6976 24	163	308·604 75	13·34	439094·0	32	2·56	0·40	41·7	37863·0724124